Some encounters between networking and information theory

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Cost of low delay

Rate distortion function for Poisson processes — Gallager 1976

Suppose a network guarantees timely delivery of packets arriving according to a Poisson process.

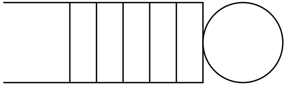
- Poisson process has rate λ, arrival times $\{A_i\}$
- Network delivers the packets at times D_i.
- Guarantee is on the delay $T_i = D_i - A_i \geq 0$:

$$\lim_n \frac{1}{n} \sum_{i=1}^{n} E[T_i] \leq d.$$

- The guarantee prohibits $\{D_i\}$ and $\{A_i\}$ being independent. The network infrastructure, in addition to what required for the packet content, must have extra capacity to 'transmit' the $\frac{1}{n} I(A^n; D^n)$ units of information in the packet timing.
- How much?

Subject to the delay guarantee, how small can $\frac{1}{n}I(A^n; D^n)$ be?

- Answer: the network needs to provision additional capacity of at least $R(\lambda d) = -\log(1 - e^{-\lambda d})$ units of information per packet ($\lambda R(\lambda d)$ per time) to meet the delay guarantee of d.
- $R(\lambda d) \approx \begin{cases} -\log(\lambda d) & \lambda d \ll 1 \\ e^{-\lambda d} & \lambda d \gg 1 \end{cases}$
- No big deal if λd is large.
- Can be significant if λd is much less than 1.



- Packets arrive to an initially empty queue at time instants $A_1, A_2, \ldots$.
- A server processes the packets, one by one, in the order of arrival.
- Servicing packet i takes S_i amount of time,
- $\{S_i\}$ i.i.d., independent of $\{A_i\}$, exponential with rate μ.
- Packets depart at times $D_1, D_2, \ldots$
- $D_i = S_i + \max\{D_{i-1}, A_i\}$.
- What can we say about $\lim \frac{1}{n} I(A^n; D^n)$?

What can we say about $\lim \frac{1}{n} I(A^n; D^n)$?

- Answer: for a given arrival rate λ

$$\frac{1}{n} I(A^n; D^n) \leq \left[\log(\mu/\lambda)\right]^+$$

  with equality iff $\{A_i\}$ is a Poisson process of rate λ.

- The capacity of the $\cdot/M/1$ queue *per time*, with the input rate λ, is

$$\lambda \log(\mu/\lambda) \quad \text{per unit time,}$$

- which is maximized at $\lambda = \mu/e$ to give

$$C = \mu \log(e)/e \quad \text{per unit time.}$$

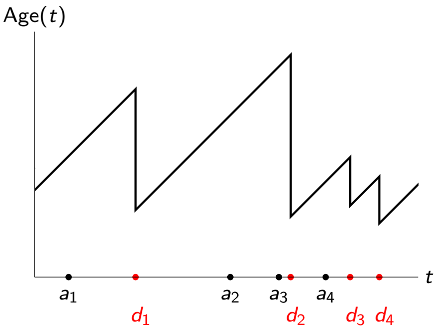
Age of information

'Low delay' does not mean 'timely':

- Even when there is no delay ($T_i = 0$), the receiver is completely up to date only at the instants departure instants D_i,
- its information getting stale while it awaits the next delivery.
- In general at a moment t between two departure instants $D_i \leq t < D_{i+1}$ the the receiver's latest information has 'age' $t - A_i$.
- If the departures are rare (i.e., if the arrivals are rare) this is a problem.

Age of information

Age for the $M/M/1$ FIFO queue — Kaul, Yates & Gruteser 2012



Simulation of the instantaneous age: queue initially empty. Black dots: arrivals, red dots: departures.

- One finds the time average to be

$$(\mu - \lambda)^{-1} + \mu^{-1}[\mu/\lambda - \lambda/\mu]$$

- For a given service rate μ the average age is minimized by choosing $\lambda \approx 0.53\mu$.

Price of youth

To guarantee freshness how much information is needed?

Take a discrete time model:

- Suppose $\{U_i : i \geq 1\}$ is a random process. (State of a system to be monitored.)
- Suppose $\{V_i : i \geq 1\}$ is a random process. (What is delivered to the monitor.)
- Causality: $U_{i+1}^{\infty} — U^i — V^i$.
- Let K_i be the largest j for which (j, U_j) may be determined from V^i.
- What is the tradeoff

$$\text{Age} = \lim \frac{1}{n} \sum_{i \leq n} E[i - K_i] \quad \text{versus} \quad \lim \frac{1}{n} I(U^n; V^n)?$$

Remarks

- Elegant problems hide in the intersection of queueing/networking/information theory.
- Even in very classical contexts (such as the $M/M/1$ queue) there are new (and interesting) questions one can pose.
- Very little has been explored in this intersection. The union of information theory and networking still remains unconsummated.